Advances in Multilevel Latent Variable Models for PISA Data

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Based on

Kaplan, D., Kim J-S., & Kim, S-Y. (2009). Multilevel Latent Variable Modeling: Current Research and Recent Developments. In R. E. Millsap and A. Maydeu-Olivares (eds.), *The SAGE Handbook of Quantitative Methods in Psychology*. Newbury Park: SAGE Publications.

- Introduction to the problem
- Multilevel latent variable modeling
- An example of multilevel factor analysis
- An example of multilevel SEM

- There are many instances of research problems in the social and behavioral sciences where observations are not simple random samples from some defined population.
- For example, the multi-stage sampling design of PISA results in data that are hierarchically structured.
- Ignoring the sampling structure through the disaggregation or aggregation of data derived from such structures is fraught with well-known problems.
- For the 20 years methodologists and statisticians have made important advances in the analysis of hierarchical data that allow for appropriate modeling of organizational systems such as schools.
- In addition, there have been recent developments that have integrated multilevel modeling with structural equation modeling (SEM) in order to provide a general methodology that can account for issues of measurement error, mediation, and simultaneity.

- For this talk, I assume that you are familiar with conventional multilevel modeling (HLM) as well as conventional factor analysis and structure equation modeling.

- Technical details of this talk can be found in

  Kaplan, D., Kim J-S., & Kim, S-Y. (2009). Multilevel Latent Variable Modeling: Current Research and Recent Developments. In R. E. Millsap and A. Maydeu-Olivares (eds.), *The SAGE Handbook of Quantitative Methods in Psychology*. Newbury Park: SAGE Publications.

- In essence, multilevel modeling is a general class of methods that allow one to model the natural clustering of observations in groups (e.g. students in schools).
    - Predictors can be added at both the student level and school level of the model.
- Factor analysis has its origins in the problem of the measurement of intelligence.
    - Factor analysis postulates a latent variable that accounts for the intercorrelations among the observed variables.
- SEM is an extension of path analysis that allows one to postulate the effects of variables that are assumed to mediate the relationship between background predictors and outcomes of interest.
    - The difference between SEM and standard path analysis, is that SEM operates at the level of the latent variable.

- It is usually straightforward to specify a factor structure for the within school (student level) variables such as attitudes toward schools or subject matter.

- It is also straightforward to allow for within school variables to vary between schools. That is, we assume that schools might account for some of the variability we observe in the student level responses.

- Conceptual difficulties sometimes arise in warranting a factor structure to explain variation between groups.

- The fact that it is sometimes difficult to conceptualize a factor structure for the between groups covariance matrix does not diminish the importance of taking the between group variability into account when conducting a factor analysis on multilevel structured data.

An Example of Multilevel Factor Analysis

- In this example, I use the South Korea sample of the PISA 2003 cycle.
- I estimate a single level and multilevel confirmatory factor analysis with and without the addition of gender as a covariate.
- On the basis of initial exploratory factor analyses, I specify two within school factors and one between school factor.
- The first within school factor can be factor labeled *CALCULATING MATHEMATICS IN LIFE* and the second within school factor can be labeled *SOLVING EQUATIONS*.
- The single between school factor can be interpreted as representing perhaps an overall school level emphasis on mathematics instruction, and can be labeled *GENERAL MATHEMATICS EMPHASIS*.
- Sometimes this between school factor takes on the look of an “average profile” for schools within a country.

- As an additional analysis, I added gender as a predictor of the latent variables with males coded 0 and females coded 1.
- Adding a predictor to a CFA model yields the specification of a multiple indicator multiple cause (MIMIC) structural equation model.

Multilevel Factor Analysis Results

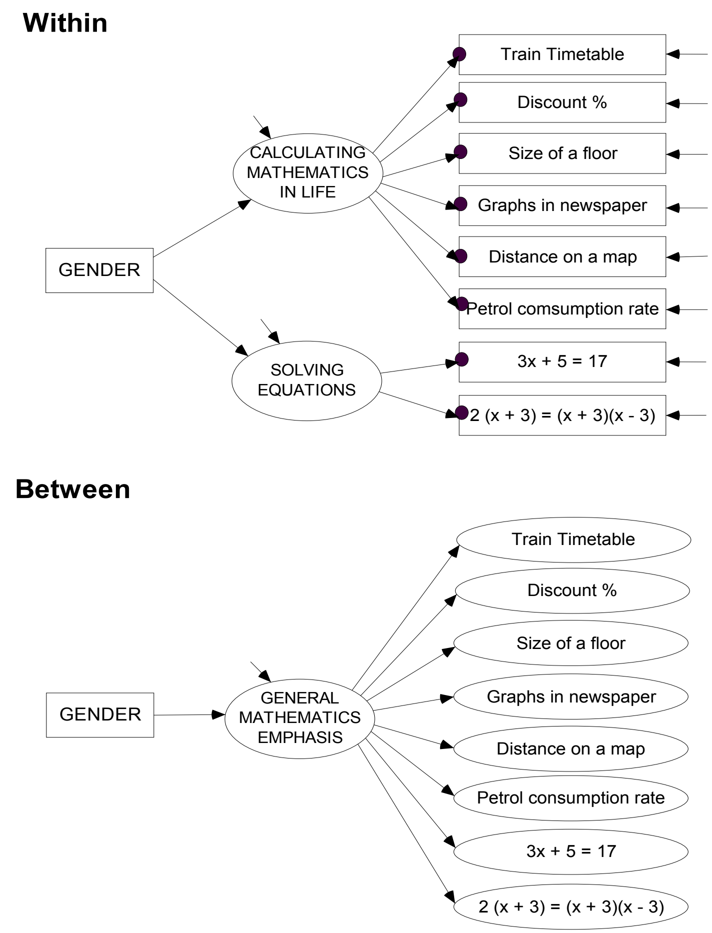

Figure 24.1. Multilevel factor analysis with a covariate.

Table 1: Results of Confirmatory Factor Analysis of PISA 2003 Mathematics Assessment

	Single-Level CFA				Multilevel CFA			
	WO Predictors		*With Predictors*		*WO Predictors*		*With Predictors*	
	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE
					Within School Model			
Calculating Mathematics								
Train timetable	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000
Discount %	1.187*	0.022	1.190*	0.022	1.136*	0.026	1.135*	0.025
Size (m^2) of a floor	1.140*	0.023	1.140*	0.023	1.125*	0.027	1.124*	0.027
Graphs in newspaper	0.909*	0.021	0.908*	0.021	0.876*	0.026	0.875*	0.026
Distance on a map	1.184*	0.028	1.185*	0.028	1.113*	0.031	1.109*	0.033
Petrol consumption rate	0.881*	0.022	0.883*	0.022	0.905*	0.027	0.904*	0.027
Solving equations								
$3x+5=17$	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000
$2(x+3)=(x+3)(x-3)$	1.060*	0.015	1.059*	0.015	1.039*	0.020	1.036*	0.021
Calculating Mathematics on MALE			-0.133*	0.016			-0.113*	0.024
Solving Equations on MALE			-0.039	0.023			0.001	0.038
Factor Covariances								
Calculating Mathematics with *Solving Equations*	0.286*	0.009	0.284*	0.009	0.197*	0.008	0.197*	0.008
					Between School Model			
General Mathematics Emphasis								
Train timetable					1.000	0.000	1.000	0.000
Discount %					1.373*	0.067	1.379*	0.068
Size (m^2) of a floor					1.192*	0.062	1.195*	0.060
Graphs in newspaper					1.047*	0.063	1.053*	0.064
Distance on a map					1.460*	0.102	1.474*	0.105
Petrol consumption rate					0.752*	0.072	0.764*	0.074
$3x+5=17$					1.808*	0.132	1.814*	0.143
$2(x+3)=(x+3)(x-3)$					1.987*	0.136	1.994*	0.145
General Mathematics Emphasis on MALE							-0.057	0.051
Model Fit Indices								
χ^2	456.250 (19 *df*)		526.500 (25 *df*)		641.253 (39 *df*)		670.784 (52 *df*)	
AIC	86593.8		95130.6		85173.1		89797.8	
BIC	86758.6		95308.9		85443.3		90088.2	

Note. Unstandardized estimates are displayed. SE: standard error. AIC: the Akaike information criterion. BIC: the Bayesian information criterion.

- Comparison of the single level and multilevel results without predictors suggests that accounting for clustering slightly worsened model fit as evidenced by the larger likelihood ratio chi-square, CFI, and RMSEA.
- The estimates are also negligibly different with the exception that the standard errors for the multilevel solution are uniformly larger.
- It should be noted that taking into account clustering is known to improve fit in simulation studies.
- In the context of real data however, accounting for clustering is still appropriate, but can also reveal other problems that can lead to poorer fit.
- In this analysis, we find that the multilevel factor model is preferred based on model selection measures - AIC and BIC.

- An important feature of the multilevel model with the predictor added is that predictors could be added at both levels.
- Interpretation would have be undertaken cautiously.
- For PISA, an important aspect of this model is that policy relevant predictors (perhaps supporting counterfactuals) could be added along with background predictors to study policy implications on latent variables (the error free measures).
- This is something that should be explored in advancing the utility of secondary analyses of PISA.

Multilevel Path Analysis

- Conventional multilevel regression models may not be suited for capturing the structural complexity within and between organizational levels.
- For example, it may be of interest to determine if school level variation in student science achievement can be accounted for by school level variables.
- Moreover, one might hypothesize and wish to test direct and indirect effects of school level exogenous variables on that portion of student level achievement that varies over schools.
- These questions are important for a fuller understanding of educational systems and such questions can be addressed via multilevel structural equation modeling.

- The model that we will consider allows for varying intercepts and varying structural regression coefficients. Earlier work on multilevel path analysis by Kaplan & Elliott (1997a) building on the work of Muthen & Satorra (1989) specified a structural model for varying intercepts only.

- This "intercepts as outcomes" model was applied to a specific educational problem in Kaplan & Elliott (1997b) and Kaplan & Kreisman (2000).

- Recent developments now allow for modeling structural slopes.

- A multilevel path analysis was employed to study within and between school predictors of mathematics achievement again using data from the PISA 2003 survey.
- The final outcome variable at the student level was a measure of mathematics achievement (MATHSCOR).
- Mediating predictors of mathematics achievement consisted of whether students enjoyed mathematics (ENJOY) and whether students felt mathematics was important in life (IMPORTNT).
- Student exogenous background variables included student's perception of teacher qualities (PERTEACH), as well as both parent's educational levels (MOMEDUC & DADEDUC).

- At the school level, a model was specified to predict the extent to which students are encouraged to achieve their full potential (ENCOURAG).
- A measure of teachers’ enthusiasm for their work (ENTHUSIA) was viewed as an important mediator variable between background variables and encouragement to make students achieve full potential.
- The variables used to predict encouragement via teachers’ enthusiasm consisted of math teachers’ use of new methodology (NEWMETHO), consensus among math teachers with regard to school expectations and teaching goals as they pertain directly to mathematics instruction (CNSENSUS), and the teaching conditions of the school (CNDITION).
- The teaching condition variable was computed from the shortage of school’s equipment, so higher values on this variable reflect a worse condition.

Multilevel Path Analysis Results

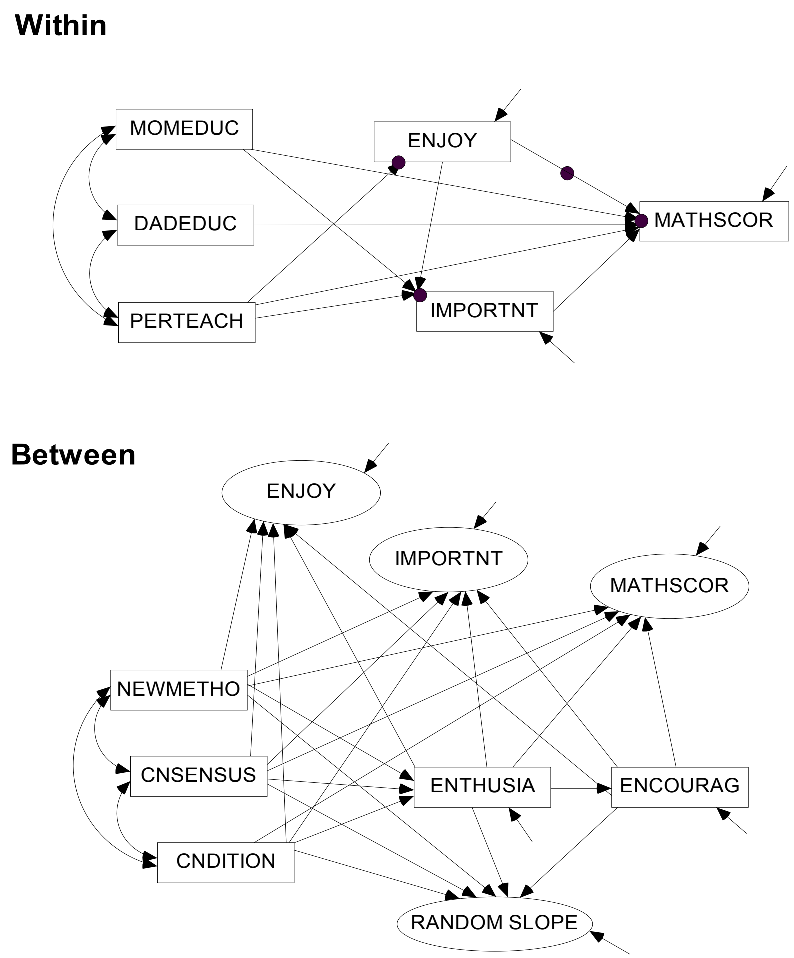

Figure 24.2. Multilevel path model of mathematics achievement with structural model at the between school level.

- First we estimated the intra-class correlations to determine the amount of variation in the student level variables that can be accounted for by differences between schools.
- We found intra-class correlations (not shown) ranging from a low of 0.02 for the importance of math in one's life to a high 0.259 for mathematics achievement.
- Under the heading "Within School" we find that MOMEDUC, DADEDUC, ENJOY, and IMPORTNT are significant and positive predictors of MATHSCOR.
- We also observe that ENJOY is significantly and positively predicted by PERTEACH. Finally, I MOMEDUC, PERTEACH, and ENJOY is a positive and significant predictor of MPORTNT.

Table 2: Results of Multilevel Path Analysis

Within School Model	Estimate.	SE	**Between School Model**	Estimate	SE
MATHSCOR on			RANDOM SLOPE on		
MOMEDUC	4.011*	1.042	NEWMETHO	-4.632	2.652
DADEDUC	4.813*	0.929	ENTHUSIA	10.101*	3.838
PERTEACH	6.273*	2.765	CNSENSUS	-3.629	3.224
IMPORTNT	15.873*	2.334	CNDITION	-8.181*	2.532
			ENCOURAG	-1.668	2.863
ENJOY on					
PERTEACH	0.457*	0.026	MATHSCOR on		
			NEWMETHO	6.806	6.550
IMPORTNT on			ENTHUSIA	-14.081	8.881
MOMEDUC	0.026*	0.006	CNSENSUS	2.407	7.898
PERTEACH	0.245*	0.021	CNDITION	3.366	6.683
ENJOY	0.534*	0.015	ENCOURAG	14.594	7.299
			ENJOY on		
			NEWMETHO	0.008	0.025
			ENTHUSIA	0.016	0.038
			CNSENSUS	0.109*	0.036
			CNDITION	0.019	0.025
			ENCOURAG	-0.035	0.024
			IMPORTNT on		
			NEWMETHO	-0.027	0.019
			ENTHUSIA	0.028	0.031
			CNSENSUS	0.057	0.030
			CNDITION	0.044*	0.020
			ENCOURAG	0.002	0.020
			ENCOURAG on		
			ENTHUSIA	0.579*	0.086
			ENTHUSIA on		
			NEWMETHO	0.164*	0.044
			CNSENSUS	0.323*	0.067
			CNDITION	-0.042	0.040

Note. Unstandardized estimates are displayed. SE: standard error.

- What is of importance to this talk are the results under the heading “Between School”.
- We find that the resource conditions of the school (CNDITION) and the extent to which the school encourages students to use their full potential (ENCOURAG) are both significant predictors of math achievement.
- Enjoyment of mathematics is significantly related to whether there is consensus among mathematics teachers in with regard to expectations and teaching goals. Importance of mathematics is related to the resource conditions of the school.
- Teacher enthusiasm for their work is significantly predicts the extent to which they encourage students to use their full potential.
- Enthusiasm is predicted by use of new methods for teaching math and the extent of consensus around school expectations and teaching goals pertaining to mathematics instruction.

- The results for the random slope relating ENJOY to MATHSCOR reveals that teacher enthusiasm moderates the relationship between enjoyment of mathematics and math achievement – with higher levels of teacher reported enthusiasm associated with a stronger positive relationship between enjoyment of math and math achievement.
- Finally, the condition of the school also demonstrates a significant moderating effect on the relationship between enjoyment of math and math achievement, where poorer conditions of the school lowers the relationship between enjoyment of mathematics and math achievement.
- The importance of this model for PISA analyses is that if between (or within) school variables could, in principle, be manipulated in the context of a hypothetical experiment, then this model could be used to test cross level causal hypotheses taking into account the structural relationships between and within levels.
- This point is related to a specific counterfactual model of causality based on manipulability theory (Woodward, 2003) and is crucial for policy analysis with PISA.

Conclusions

- This presentation demonstrated the application of multilevel factor analysis and multilevel SEM with PISA data.
  - I didn't account for the complex sampling design but that is quite easily done with Mplus.
- This application touches on only a small portion of what can be done in the multilevel setting.
- Extensions include...
  - Latent class and multilevel latent class models
  - Mixture multilevel factor analysis and multilevel SEM

- The relevance of these models for the analysis of PISA data cannot be over-stated.
- Nevertheless, support for causal inferences cannot be read directly off a path diagram without assumptions that are not directly part of the statistical modeling method.
- In particular (and in my opinion), these models can be used to read off causal statements if the variables themselves support counterfactual thinking, as in the Rubin Causal Model (Rubin, 1974; Holland, 1986)
- We need to conceive of an unit (student or school) as beginning in both the “treatment” and “control”.
- Our ability to warrant causal inferences along these lines depends on how relevant policy variables are measures.
- My hope is that we can push the utility of PISA by directly engaging in policy relevant simulations on statistical models with variables that support counterfactual causal hypotheses.

THANK YOU